

PREPRINTS

17 papers

An overview of the papers released at github.com/suvrit/preprints. Except where marked, the mathematics in these papers was developed in sustained dialogue with an AI assistant (GPT Pro); none of the results is zero-shot. Claude Code was used extensively for writing, editing, and repository management. Results are grouped by subject; entry numbers do not indicate a ranking. Each summary is followed by the paper's title, linked to its latest timestamped PDF, together with its version and, where one exists, its arXiv identifier.

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COMBINATORICS

1. McLeod's 1959 conjecture. The paper proves McLeod's conjecture that $(h_{\ell+r}/h_\ell)^{1/r}$ is subadditive for all $\ell \geq 0$ and $r \geq 1$, where h_k is the complete homogeneous symmetric polynomial of degree k . The first step shows, via a closed queueing network, that h_k/h_{k-1} is convex; a tensorization argument then couples neighboring ratios tightly enough to make their geometric mean convex.

A Proof of McLeod's 1959 Conjecture on Complete Homogeneous Symmetric Ratios · v1, 5 October 2026 · arXiv:2610.07302

2. Monotone Turán-ratios of Schur polynomials. The paper proves that, unless all n padded parts of λ are equal, the Turán-ratio $S_{\lambda,i}(x)^2/(S_{\lambda,i-1}(x)S_{\lambda,i+1}(x))$ is strictly increasing for $x > 1$ and $1 \leq i < n$, where $S_{\lambda,i}(x) = s_\lambda(1^{[n-i]}, x^{[i]})$ is the Schur polynomial in n variables with i arguments equal to x and the rest equal to 1. For one-row shapes this answers a question of Ait-Haddou on complete homogeneous symmetric polynomials; in that case a second proof, through the zeros of Meixner polynomials, gives an explicit positive formula for the logarithmic derivative of the ratio. It conjectures the analogue for Macdonald polynomials.

Monotonicity of Turán-ratios of symmetric polynomials · v1, 7 October 2026

3. A Kotljanskii inequality for permanents. The paper proves that $\text{per}(A_{S \cup T}) \text{per}(A_{S \cap T}) \geq \text{per}(A_S) \text{per}(A_T)$ for all $S, T \subseteq [n]$ when A is an inverse M -matrix that becomes symmetric after a positive diagonal similarity; here A_S is the principal submatrix indexed by S . The proof writes permanents as moments of a complex Gaussian vector and applies Ginibre's correlation inequality. An explicit counterexample shows that symmetry cannot be dropped.

A Kotljanskii inequality for permanents · v1, 2 October 2026 · arXiv:2609.39979

4. Majorization for normalized Jack and Macdonald polynomials. The paper proves that if λ majorizes μ , then the Jack polynomial P_λ in n variables, normalized to equal one at $(1, \dots, 1)$, is at least the normalized P_μ on the positive orthant, for every n , and strictly so when the coordinates are distinct. For the conjectured Macdonald analogue, it proves the rank-two case, the case of a one-row partition against the hook below it for $t \leq q$ in every rank, and, by exact certificates, the full statement in three variables for partitions of size at most eight. It shows that the Harish-Chandra–Itzykson–Zuber mechanism behind the Schur and Jack proofs has no direct finite- q analogue. The full conjecture remains open.

On majorization inequalities for normalized Jack and Macdonald polynomials · v1, 8 October 2026

5. A fusion inequality for Macdonald plethysms. The paper proves that the plethysm $H_m[\tilde{H}_\mu]$ of a modified Macdonald polynomial by the modified Hall–Littlewood polynomial H_m lies above the Macdonald polynomial $\tilde{H}_{m\mu}$ obtained by fusing m copies of $\tilde{H}_\mu$: the quotient $(H_m[\tilde{H}_\mu] - \tilde{H}_{m\mu})/(1 - q^m)$ is Schur positive with coefficients in $\mathbb{N}[q, t]$ and is the character of an explicit module, the fusion defect. A symmetry of the modified Macdonald polynomials reverses this comparison at the cost of a degree shift, whose least value the paper determines. For one-row shapes the two comparisons give a shifted form of Bergeron’s q -analogue of Foulkes’ conjecture: $(H_b[H_a] - q^{a(a-1)\lfloor b/2 \rfloor} H_a[H_b])/(1 - q)$ is Schur positive for all $a, b \geq 2$, where the conjecture asks for this without the factor $q^{a(a-1)\lfloor b/2 \rfloor}$ when $a \leq b$. A shift linear in b also holds for Bergeron’s rectangular version.

The q -Foulkes problem I: A fusion inequality for Macdonald plethysms · v1, 26 October 2026



MATRIX ANALYSIS

6. Lin’s singular-value conjecture. The paper proves Lin’s conjecture that $\lambda_j((1 - t)A + tB) \geq \sigma_j^{1/2}(A^{2(1-t)}B^{2t})$ for all positive semidefinite A, B , all j , and every $t \in [0, 1]$. The case $t = \frac{1}{2}$ is Drury’s theorem, which answered a question of Bhatia and Kittaneh. The proof follows Drury’s argument but replaces its block-matrix step, which works only at the midpoint, with an eigenvalue count based on an inertia identity and a one-variable inequality.

On Lin’s Arithmetic–Geometric Means Conjecture for Singular Values · v1, 8 October 2026

7. Bellman’s 1959 claim on Hua–Bellman matrices. The paper shows that Bellman’s 1959 strengthening of Hua’s results on positive definiteness of the matrices $[\det(I - A_i^* A_j)^{-\alpha}]_{ij}$, for strict contractions A_i , rests on an erroneous proof and, by an explicit example, that it is false. It gives conditions under which $\det(I - A^* B)^{-\alpha}$ is a positive definite function: for complex contractions, the known sharp condition, with the integer exponents below Hua’s range reached by an elementary argument; for real symmetric contractions, the part of Bellman’s claim that survives. It also introduces a Poincaré-like distance on noncommuting strict contractions.

Positive definite functions of noncommuting contractions, Hua–Bellman matrices, and a new distance metric · v1, 2 October 2026 · arXiv:2112.00056v2 · Not AI-assisted

8. The p -Bures–Wasserstein distance and the Lin–van den Driessche conjecture. The paper proves that $D_p(A, B) = (\text{tr } A^p + \text{tr } B^p - 2\|A^{1/2}B^{1/2}\|_p^p)^{1/2}$, which is the Bures–Wasserstein distance at $p = 1$ and the Hilbert–Schmidt distance at $p = 2$, is a metric on the positive semidefinite matrices for every $1 \leq p \leq 2$. It does so by proving the 2014 conjecture of Lin and van den Driessche that $[\|H_{ij}\|_p^p]_{i,j=1}^3$ is positive semidefinite for every positive semidefinite 3×3 block matrix $[H_{ij}]$; the proof is independent of that of Vuong and Osaka, who proved both results by a different method. The main tool is a partition inequality for Schatten norms, proved by complex interpolation. The same inequality gives sufficient decrease for a fixed-point iteration that computes the barycenters of D_p ; for positive definite data the iteration converges linearly from every positive definite start.

The p -Bures–Wasserstein distance via 3×3 block matrices (with application to p -Bures–Wasserstein barycenters) · v1, 11 October 2026 · arXiv:2610.15889

9. Tsallis metrics on density matrices. The paper proves that $d_\alpha(\rho, \sigma) = |2 - \|\rho^\alpha + \sigma^\alpha\|_{1/\alpha}|^{1/2}$ is a metric on density matrices for $1/2 \leq \alpha \leq 2$, $\alpha \neq 1$; up to the factor $|1 - \alpha|$, d_α^2 is the least total Tsallis relative entropy of order α from two states to a common state, and $\alpha \uparrow 1$ recovers the quantum Jensen–Shannon metric. For $\alpha < 1$ the main step shows that $[-\log \det(I - U_i U_j)]$ is positive semidefinite for any three Hermitian strict contractions U_1, U_2, U_3 ; equivalently, the S-divergence is of negative type on any four positive definite matrices, and four is sharp. For $\alpha > 1$ the paper proves the endpoint $\alpha = 2$ and transfers it to the other orders by tensoring.

Tsallis Metrics on Density Matrices via the S-Divergence · v1, 13 October 2026

10. A spectral Blakley–Roy inequality for nonnegative matrices. The paper proves that $\rho(AB)^k \leq \rho(A^k B^k)$ for all real symmetric matrices A, B with nonnegative entries and every integer $k \geq 1$, where ρ is the spectral radius; for $B = uu^T$ this is the Blakley–Roy inequality, and the general case was conjectured by Pourya Habibzadeh. The route is the spectral-norm inequality $\|(AB)^m\|^k \leq \|(A^k B^k)^m\|$ for all integers $k, m \geq 1$: even k follows from Cordes’ inequality, and odd $k = 2r + 1$ from the product inequality $(u^T C_1 \cdots C_r v)^k \leq (u^T u)^r (v^T v)^r u^T C_1^k \cdots C_r^k v$ for nonnegative symmetric C_i and nonnegative u, v , proved by an entropy argument on paths that traverse each edge k times, back and forth; its exact remainder characterizes equality.

A Spectral Blakley–Roy Inequality for Nonnegative Matrices · v1, 15 October 2026



COMPLEX GEOMETRY

11. Schur forms under decomposable curvature. Griffiths asked in 1969 whether positive curvature of a Hermitian holomorphic vector bundle forces positivity of its characteristic forms; Du’s rank-nine example shows that a strictly Griffiths-positive metric can have a top Chern form that is negative on an open set. The paper proves that every Schur form is weakly nonnegative wherever the curvature is decomposable, that is, a sum of a completely positive map and a completely copositive map, with no splitting assumption. The algebraic input is the inequality $\Phi_n(H) \geq 0$ for Finski’s double mixed discriminant of every decomposable map H , obtained from a quaternionic Gaussian determinant formula. Beyond the decomposable cone, $\Phi_k \geq 0$ on every square compression of H is equivalent to $\Phi_n(H + S) \geq 0$ for every entanglement-breaking map S ; the class of maps satisfying this condition contains indecomposable maps but is not convex for $n \geq 9$, since Du’s map is an average of four of its members.

Weak positivity of Schur forms under decomposable curvature · v1, 8 October 2026

12. Lower bounds for the quantum permanent. Du’s nine-dimensional example shows that the quantum permanent Φ_n , Finski’s double mixed discriminant, can be negative on positive bistochastic maps. The paper bounds how negative. The orbital polynomial of the map turns Φ_n into a linear function of a probability distribution on partitions, and positivity of the map into linear constraints on that distribution, which come from compressions to smaller dimensions and from the spectra of projection images. These give $\Phi_n \geq -3n/(3n^2 + 20n + 103)$ for $n \geq 8$ and $\Phi_n \geq -1/n + (3e^2 - 7 - o(1)) \log n/n^2$, and $3e^2 - 7$ is the best coefficient the constraints can certify recursively. Explicit maps in dimensions eight and nine improve on Du’s example; padding them gives $-\mu_n = \Omega(n^{-8})$, and the order is conjectured to be n^{-d_*} for the first negative dimension d_* .

Lower bounds for the quantum permanent · v1, 24 October 2026



MATHEMATICAL PHYSICS

13. Petz’s conjecture on the Kubo–Mori scalar curvature. The paper proves Petz’s 1994 conjecture that the scalar curvature of the Kubo–Mori metric on the invertible density matrices increases as the state becomes more mixed. Dittmann (2000) reduced the conjecture to the concavity of an explicit symmetric function h of three positive variables, which he could support only by numerical tests; the paper proves that h is strictly concave. The proof writes $h = -1/(8M)$ with M positive and homogeneous of degree one, and shows that $1/M$ is a Stieltjes function along every line with positive base point and positive direction; Cauchy–Schwarz and homogeneity then give concavity, and the Stieltjes property reduces to an explicit inequality in two real variables, proved by elementary estimates for hyperbolic functions.

A Proof of Petz’s Conjecture on the Scalar Curvature of the Kubo–Mori Metric · v1, 9 October 2026

14. The Gibilisco–Isola conjecture near the Kubo–Mori metric. The paper proves the Gibilisco–Isola conjecture that the scalar curvature of the Wigner–Yanase–Dyson metric with parameter p on the invertible density matrices increases under mixing for all p in some interval $(1, 1 + \varepsilon)$, together with the mirror statement for $p \in (1 - \varepsilon, 1)$, with $\varepsilon > 0$ independent of the dimension; the case $p \downarrow 1$ is Petz’s conjecture, result 13. For these p the curvature is a concave function of the eigenvalues. By Dittmann’s formula it suffices that one symmetric function H_p of three positive variables is concave, which the paper proves by showing that $-1/H_p$ is a Stieltjes transform along every line in the positive cone, uniformly in p near 1; the uniformity near the boundary of the eigenvalue domain comes from an exact decomposition of H_p . The value of ε is not explicit.

Wigner–Yanase–Dyson Metrics Have Monotone Curvature · v1, 9 October 2026

REAL AND COMPLEX ANALYSIS

15. Reverse regularity of the square-root theta kernel and Csordas’s conjecture. The paper proves that the Hankel kernel $K(x, y) = \Phi(\sqrt{x+y})$, where Φ is the Jacobi theta kernel in the Fourier representation of Ξ , is strictly reverse-regular through order four, and that order four is sharp. The coefficient minors give ordinary, double, and triple Turán inequalities, and Fourier inversion gives global multilinear integral inequalities for Ξ . The third-order sign gives an independent proof of Csordas’s conjecture that $s'^2 - ss''$ is strictly log-concave for $s(t) = \Phi(\sqrt{t})$. The proof combines exact polynomial estimates on the tail with interval computations.

Sharp order-four sign regularity of a Jacobi theta kernel · v1, 10 October 2026

16. Log-concavity and Pólya frequency of order ten for $1/\Xi$. The paper proves strict log-concavity of the positive functions Θ and Λ whose one-sided and bilateral Laplace transforms are $1/\xi(\frac{1}{2} + \sqrt{s})$ and $1/\Xi$; for either function, log-concavity is the Pólya frequency property of order two, and the property of every order is equivalent to the Riemann hypothesis. Without assuming the Riemann hypothesis, but using the verification of the zeros through height $3 \cdot 10^{12}$, Λ is a Pólya frequency function of order ten, and for each fixed order the ordered minors of $\Theta(x - y)$ are positive when all differences $x_i - y_j$ are small and positive. Also proves the generalized Laguerre inequalities $L_n[\Xi](x) \geq 0$ for every $n \geq 0$ and $|x| \leq T - \frac{1}{2}$ whenever the Riemann hypothesis holds up to height T ; the case $n = 1$ is due to Csordas, Ruttan and Varga.

On some positivity properties of the Riemann Ξ function · v1, 20 October 2026

PROBABILITY

17. A determinant inequality for matrix Gaussian series. The paper proves that if $X = \sum_i g_i B_i$, where the g_i are independent standard Gaussians and the B_i are real symmetric $d \times d$ matrices with $\sum_i B_i^2 \leq I_d$, at most d of them linearly independent, then $\mathbb{E} \det f(X) \geq (\mathbb{E} f(13G/5))^d$ for every even log-concave $f: \mathbb{R} \rightarrow [0, \infty)$, where G is a standard Gaussian. Interval indicators give the small-ball bound $\mathbb{P}(\|X\| \leq 13a/5) \geq \mathbb{P}(|G| \leq a)^d$ and the hereditary small-ball bound needed by partial coloring for matrix Spencer discrepancy. The proof reduces each function to one small-ball estimate, obtained from matrix-weighted Poincaré inequalities, and controls the rest by the averaged comparison $\mathbb{E} \operatorname{tr} \psi(|X|) \leq d \mathbb{E} \psi(\sqrt{\pi/2} |G|)$ for convex nondecreasing ψ with $\psi(0) = 0$, whose constant is optimal. Anticommuting coefficients show that no universal dilation below $\sqrt{\pi/2}$ works; the paper conjectures that $\sqrt{\pi/2}$ is optimal, gives an equivalent entropy formulation, and proves the inequality at $\sqrt{\pi/2}$ for $f(x) = e^{-a|x| - bx^2}$ when at most $7d/8$ coefficients are linearly independent, and for all functions under an ellipsoidal norm bound in large dimension.

Beyond Small Balls: A Determinant Inequality for Functions of Matrix Gaussian Series · v1, 19 October 2026



See **12. Lower bounds for the quantum permanent** under Complex geometry.

